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# **Earthquake Analysis and Design of Simple Structures and Equipment: *Response Spectrum Method***

Course No: S01-002  
Credit: 1 PDH

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## **1. SCOPE**

The course presents the response spectrum method of earthquake response analysis of simple structures and mechanical/electrical equipment. The method presented in the course is widely used and is applicable to any structure or equipment that can be modeled as a single-degree-of-freedom (SDOF) system. No prior knowledge of structural vibration analysis or earthquake analysis is required to understand this course.

## **2. INTRODUCTION**

Three methods of earthquake response analysis of simple structures and equipment modeled as single degree of freedom (SDOF) system are available.

1. Simplified procedures such as those provided in ASCE 7 Standard [1]
2. SDOF response spectrum method
3. Time-history method

The SDOF response spectrum method can be used as long the structure or equipment can be modeled by a single degree of freedom system consisting of a spring, mass and damping. The SDOF response spectrum method usually gives less conservative results than simplified procedures, yet easy to perform. It is far easier to perform than time-history analysis that requires structural dynamic analysis computer software. Time history analysis provides both the time history of the response (for example, displacement, velocity and acceleration) and maximum value of the response. Response spectrum method provides only the maximum value of response. (Plot of response versus time is called response time history).

### 3. SDOF MODEL

A single degree of freedom system is shown in Figure 1.

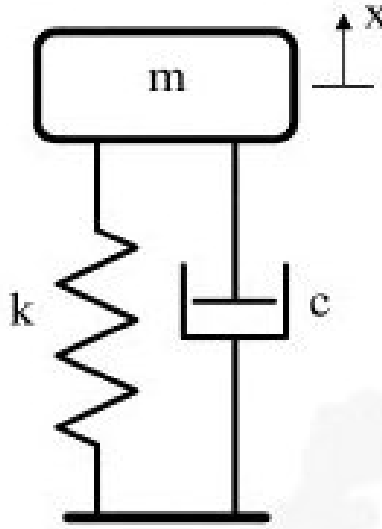


Figure 2: Single-Degree-of-Freedom System Model

$c$  = damping,  
 $k$  = stiffness,  
 $m$  = mass, and  
 $x$  = displacement of the mass

$k$  = stiffness of supports in the direction of the earthquake motion (If a horizontal earthquake motion is considered, then  $k$  is the corresponding horizontal stiffness of the supports; if vertical earthquake motion is considered, then  $k$  is the vertical stiffness of the supports.)

$$m = w/g \quad (1)$$

where  $w$  = weight of SDOF  
 $g$  = acceleration due to gravity

Damping is often specified as percentage of critical damping, or simply damping percentage, and denoted by  $\xi$ . As an example, 2% damping is often used for welded steel construction in some industries. Damping may also be presented as damping ratio, again denoted by  $\xi$ . If percentage of critical damping is 2%, then damping ratio is 0.02.

#### 4. NATURAL FREQUENCY AND PERIOD OF THE SDOF SYSTEM

Natural frequency of the SDOF system in radians per second is given by

$$\omega^2 = k/m \quad (2)$$

Natural frequency in cycles per second is given by

$$f = \omega/2\pi \quad (3)$$

Cycles/second is also known as hertz.

Period of the SDOF system in seconds is given by

$$T = 1/f \quad (4)$$

#### 5. RESPONSE SPECTRUM

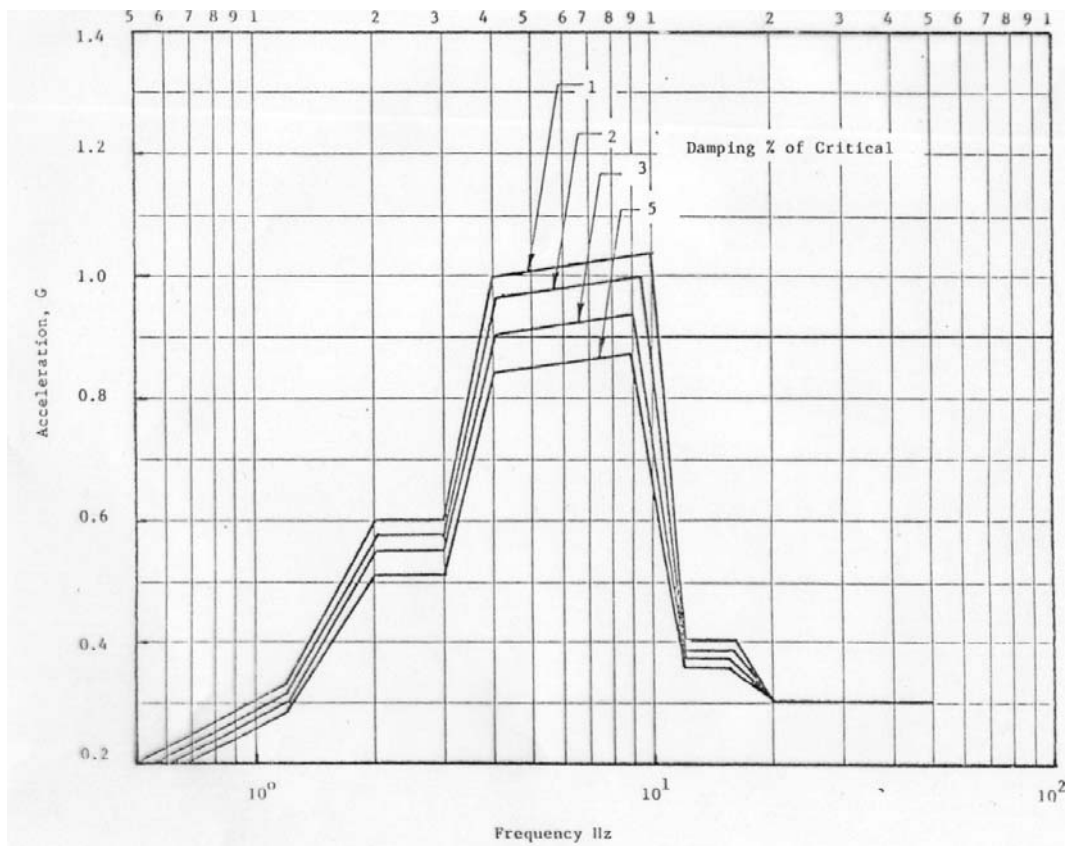


Figure 2: Sample acceleration response spectra

A family of acceleration response spectra for various damping values is shown in Figure 2. Horizontal axis is the frequency in hertz (hz) and vertical axis is the maximum acceleration of the mass as a multiple of g. This family of response spectra represents a given base motion of the SDOF system. (Note: Sometimes acceleration response spectra are plotted as "Period versus maximum acceleration" instead of "frequency versus maximum acceleration". Velocity response spectra and displacement response spectra are also sometimes used. In those cases the vertical axis is maximum velocity or maximum displacement, as appropriate.)

Consider two damping values  $\xi_1$  and  $\xi_2$  where  $\xi_1 < \xi_2$ . For a given earthquake motion,  $\xi_1$  response spectrum curve will be higher than or equal to the  $\xi_2$  response spectrum curve for all values damping. This can be seen in Figure 2.

If the structure or equipment is supported at the ground level, then the ground acceleration spectra are used in the analysis. There will be separate families of horizontal and vertical spectra corresponding to the horizontal and vertical earthquake motions.

If the equipment is located at a higher floor of a building, then the response spectra for the earthquake motion at that floor are used in the analysis. Such spectra are called "in-structure response spectra" or "floor response spectra". Such spectra are usually provided by the architect-engineer or seismic structural analyst of the building.

## 6. RESPONSE COMPUTATION

### 6.1 Maximum Acceleration

The maximum acceleration experienced by the mass (in the direction of the earthquake motion) is read from the acceleration response spectra curves. The procedure will be illustrated in the solved example problem (Section 8).

### 6.2 Maximum Velocity and Displacement

Maximum velocity and displacement may be determined from the velocity and displacement response spectra, if available.

Alternately, maximum velocity and displacement may be computed from the maximum acceleration using the following approximate equations.

$$\text{Maximum relative velocity} = \text{Maximum acceleration}/\omega \quad (5)$$

$$\text{Maximum relative displacement} = \text{Maximum velocity}/\omega \quad (6)$$

Maximum acceleration in Equation. 5 should be specified in inch/sec<sup>2</sup> or feet/sec<sup>2</sup> (not as a multiple of g).

The above velocity (in inch/sec or feet/sec) and displacement (in inch or feet) are relative to the base.

### 6.3 Maximum Force

Maximum force on the mass may be computed by multiplying mass by the maximum acceleration. Maximum acceleration should be specified in inch/sec<sup>2</sup> or feet/sec<sup>2</sup>.

$$\text{Maximum force} = \text{mass} \times \text{maximum acceleration} \quad (7)$$

## 7. LOAD COMBINATION

If the design code or design guide requires that responses from more than one direction of the earthquake motion need to be combined, then each individual response is computed separately as described above and then the responses are combined per design code/guide.

## 8. NUMERICAL EXAMPLE

### Problem:

A structure weighs 4040 pounds. Support stiffness in the direction of earthquake motion is 50000 pounds/inch. Damping is 5%. The earthquake motion is represented by the response spectra shown in Figure 2. Compute the maximum acceleration, velocity, displacement and force in the direction of the earthquake motion.

### Solution:

$$\text{Weight } w = 4040 \text{ lb}$$

$$g = 386.4 \text{ inch/sec}^2$$

$$\text{Mass } m = 4040/386.4 = 10.45 \text{ lb-sec}^2/\text{inch}$$

$$\text{Stiffness} = 50000 \text{ lb/inch}$$

Using Equation 2,

$$\omega^2 = k/m = 50000 / 10.45 = 4784$$

Natural frequency  $\omega = 69.2$  radians/second

Per Equation 3, natural frequency in cycles/sec (hertz) is

$$f = \omega/2\pi = 69.2/2\pi = 11 \text{ hertz}$$

Per Equation 4, period is

$$T = 1/f = 1/11 = 0.09 \text{ sec}$$

Percentage of critical damping = 5%

Maximum acceleration is obtained from Figure 2. Go to 11 hertz on the horizontal axis. Draw a vertical line and go up to the 5% response spectra. The maximum acceleration thus read is equal to 0.6g.

$$\text{Maximum acceleration} = 0.6g = 0.6 \times 386.4 = 231.8 \text{ inch/sec}^2$$

Using Equations 5, 6 and 7,

$$\text{Maximum relative velocity} = 231.8/69.2 = 3.35 \text{ inch/sec}$$

$$\text{Maximum relative displacement} = 3.35/69.2 = 0.05 \text{ inch}$$

$$\text{Maximum force} = 10.45 \times 231.8 = 2422 \text{ lb}$$

## **9. REFERENCE**

1. "Minimum Design Loads for Buildings and Other Structures", ASCE/SEI 7, American Society of Civil Engineers.